Solutions to Workbook-2 [Mathematics] | Permutation & Combination

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39. Suppose, some square has the maximum among the (mn) natural numbers in all boxes. Since this number is the arithmetic mean of numbers in all its neighbouring squares, and all these numbers can't exceed the maximum number, all the numbers in the neighbouring square will be equal to the number in the square containing the maximum number. Now extending this argument, we get the neighbours of all the neighbours of the square containing the maximum number will also have the same number. Thus, we can prove that the entire grid will have the same number.

40.(n = 9 and r = 3)

We know that, $\dfrac{{}^nC_r}{{}^nC_{r-1}} = \dfrac{n-r+1}{r}$

$\Rightarrow \quad \dfrac{84}{36} = \dfrac{7}{3} = \dfrac{n-r+1}{r}$ [given]

$\Rightarrow \quad 3n - 10r + 3 = 0$ ...(i)

Also given, $\dfrac{{}^nC_r}{{}^nC_{r+1}} = \dfrac{84}{126} \quad \Rightarrow \quad \dfrac{r+1}{n-r} = \dfrac{2}{3} \quad \Rightarrow \quad 2n - 5r - 3 = 0$ ...(ii)

On solving Eqs. (i) and (ii), we get $r = 3$ and $n = 9$

41.(7) Reducing the equation to a newer equation, where sum of variables is less. Thus, finding number of arrangements becomes easier.

As, $n_1 \ge 1, n_2 \ge 2, n_3 \ge 3, n_4 \ge 4, n_5 \ge 5$

Let $n_1 - 1 = x_1 \ge 0, n_2 - 2 = x_2 \ge 0, \ldots, n_5 - 5 = x_5 \ge 0 \Rightarrow$ New equation will be

$x_1 + 1 + x_2 + 2 + \ldots + x_5 + 5 = 20 \quad \Rightarrow \quad x_1 + x_2 + x_3 + x_4 + x_5 = 20 - 15 = 5$

Now,

$x_1 \le x_2 \le x_3 \le x_4 \le x_5$

x_1	x_2	x_3	x_4	x_5
0	0	0	0	5
0	0	0	1	4
0	0	0	2	3
0	0	1	1	3
0	0	1	2	2
0	1	1	1	2
1	1	1	1	1

So, 7 possible cases will be there.

42.(B)

Objects	Groups	Objects	Groups
Distinct	Distinct	Identical	Identical
Distinct	Identical	Identical	Distinct

Description of Situation Here, 5 distinct balls are distributed amongst 3 persons so that each gets at least one ball. i.e. Distinct $\rightarrow$ Distinct

So, we should make cases

Case I $\left.\begin{matrix} A & B & C \\ 1 & 1 & 2 \end{matrix}\right\}$ **Case II** $\left.\begin{matrix} A & B & C \\ 1 & 2 & 2 \end{matrix}\right\}$

Number of ways to distribute 5 balls $= \left({}^5C_1 \cdot {}^4C_1 \cdot {}^3C_3 \times \dfrac{3!}{2!}\right) + \left({}^5C_1 \cdot {}^4C_2 \cdot {}^2C_2 \times \dfrac{3!}{2!}\right)$

$= 60 + 90 = 150$

43. ${}^{\frac{1}{2}(2n-k^2+k-2)}C_{k-1}$

The number of solutions of $x_1 + x_2 + \ldots + x_k = n$

= Coefficient of t^n in $(t+t^2+t^3+\ldots)(t^2+t^3+\ldots)\ldots(t^k+t^{k+1}+\ldots)$

= Coefficient of t^n in $t^{1+2+\ldots+k}(1+t+t^2+\ldots)^k$

Now, $1+2+\ldots+k = \dfrac{k(k+1)}{2} = p$ [say]

and $1+t+t^2+\ldots = \dfrac{1}{1-t}$

Thus, the number of required solutions = Coefficient of t^{n-p} in $(1-t)^{-k}$

= Coefficient of t^{n-p} in $\left[1 + {}^kC_1 t + {}^{k+1}C_2 t^2 + {}^{k+2}C_3 t^3 + \ldots\right] = {}^{k+n-p-1}C_{n-p} = {}^rC_{n-p}$

Where, $r = k+n-p-1 = k+n-1-\dfrac{1}{2}k(k+1)$

$= \dfrac{1}{2}(2k+2n-2+k^2-k) = \dfrac{1}{2}(2n-k^2+k-2)$

44. Here, n^2 objects are distributed in n groups, each group containing n objects.

$\therefore$ Number of arrangements

$= {}^{n^2}C_n \cdot {}^{n^2-n}C_n \cdot {}^{n^2-2n}C_n \cdot {}^{n^2-3n}C_n \cdot {}^{n^2-2n}C_n \ldots {}^nC_n$

$= \dfrac{(n^2)!}{n!(n^2-n)!} \cdot \dfrac{(n^2-n)!}{n!(n^2-2n)!} \ldots \dfrac{n!}{n! \cdot 1} = \dfrac{(n^2)!}{(n!)^n}$

$\Rightarrow$ Integer (as number of arrangements has to be integer)

45. **(i)** $\dfrac{(52)!}{(13)^4}$ **(ii)** $\dfrac{(52)!}{4!(13!)^4}$ **(iii)** $\dfrac{(52)!}{3!(17)^3}$

(i) The number of ways in which 52 cards be dividend equally among four players in order

$= {}^{52}C_{13} \times {}^{39}C_{13} \times {}^{26}C_{13} \times {}^{13}C_{13} = \dfrac{(52)!}{(13!)^4}$

(ii) The number of ways in which a pack of 52 cards can be divided equally into four groups of 13 cards each $= \dfrac{{}^{52}C_{13} \times {}^{39}C_{13} \times {}^{26}C_{13} \times {}^{13}C_{13}}{4!} = \dfrac{(52)!}{4!(13!)^4}$

(iii) The number of ways in which a pack of 52 cards be divided into 4 sets, three of them having 17 cards each and the fourth just one card $= \dfrac{{}^{52}C_{17} \times {}^{35}C_{18} \times {}^{18}C_{17} \times {}^1C_1}{3!} = \dfrac{(52)!}{3!(17!)^3}$.

46.(C) Number of derangements of $6 = 6!(1 - 1/1! + 1/2! - 1/3! + 1/4! - 1/5! + 1/6!)$

$= 360 - 120 + 30 - 6 + 1 = 265$

Out of these derangements, there are five ways envelopes to which card number 1 is assigned and all such derangements are equal in number.

So, it is going in envelope numbered 2 in $265/5 = 53$ ways.

47.(A) Derangement of S_2, S_3, S_4, S_5 is $= 4!\left(1 - \dfrac{1}{1!} + \dfrac{1}{2!} - \dfrac{1}{3!} + \dfrac{1}{4!}\right) = 9$

Probability $= \dfrac{9}{5!} = \dfrac{9}{120} = \dfrac{3}{40}$

48.(C) $\underbrace{5! - {}^4C_1 \times 4! \times 2!}_{\text{at least one pair}} + \underbrace{\left({}^3C_1 \times 3! \times 2! + {}^3C_1 \times 3! \times 2! \times 2!\right)}_{\text{at least two pair}} - \underbrace{\left({}^2C_1 \times 2! \times 2! + {}^2C_1 \times 2 \times 2 \times 2!\right)}_{\text{at least 3}} + 2 = 14$

Required probability $= \frac{14}{5!} = \frac{7}{60}$

49.(625) Last two-digit number divisible by 4 from (1, 2, 3, 4, 5) are 12, 24, 32, 44, 52
The number of 5-digit numbers which are divisible by 4 from the digit (1, 2, 3, 4, 5) and digit is repeated is 5 × 5 × 5 × (5 × 1) = 625

50.(C) Given 6 boys $M_1, M_2, M_3, M_4, M_5, M_6$ and 5 *girls* G_1, G_2, G_3, G_4, G_5

(A) $^6C_3 \times {}^5C_2 = 200$

(B) $^6C_1\ {}^5C_1 + {}^6C_2\ {}^5C_2 + {}^6C_3\ {}^5C_3 + {}^6C_4\ {}^5C_4 + {}^6C_5\ {}^5C_5 = 461$

(C) $^5C_2\ {}^6C_3 + {}^5C_3\ {}^6C_2 + {}^5C_4\ {}^6C_1 + {}^5C_5\ {}^6C_0 = 381$

(D) G_1 is included $\rightarrow {}^4C_1 . {}^5C_2 + {}^4C_2 . {}^5C_1 + {}^4C_3 = 74$

M_1 is included $\rightarrow {}^4C_2 . {}^5C_1 + {}^4C_3 = 30 + 4 = 34$

G_1 and M_1 both are not included $^4C_4 + {}^4C_3 . {}^5C_1 + {}^4C_2 . {}^5C_2 = 81$

Total number = 74 + 34 + 81 = 189

Now, $P \rightarrow 4; Q \rightarrow 6; R \rightarrow 5; S \rightarrow 2$